Each constellation pt has energy $= \mathcal{E}_s$.

$$s_k = \left(\underbrace{\sqrt{\mathcal{E}_s}\cos\left(\frac{2\pi k}{M}\right)}_{a_1}, \underbrace{\sqrt{\mathcal{E}_s}\sin\left(\frac{2\pi k}{M}\right)}_{a_2} \right)$$

Receiver Processing For M-ary PSK

$$x(t) = a_1 p_1(t) + a_2 p_2(t)$$

$$y(t) = x(t) + n(t)$$

$n(t)$: AWG Gaussian Noise

$$y(t) = a_1 p_1(t) + a_2 p_2(t) + n(t)$$

Energy of Pulse $\mathcal{E}_p = 1$

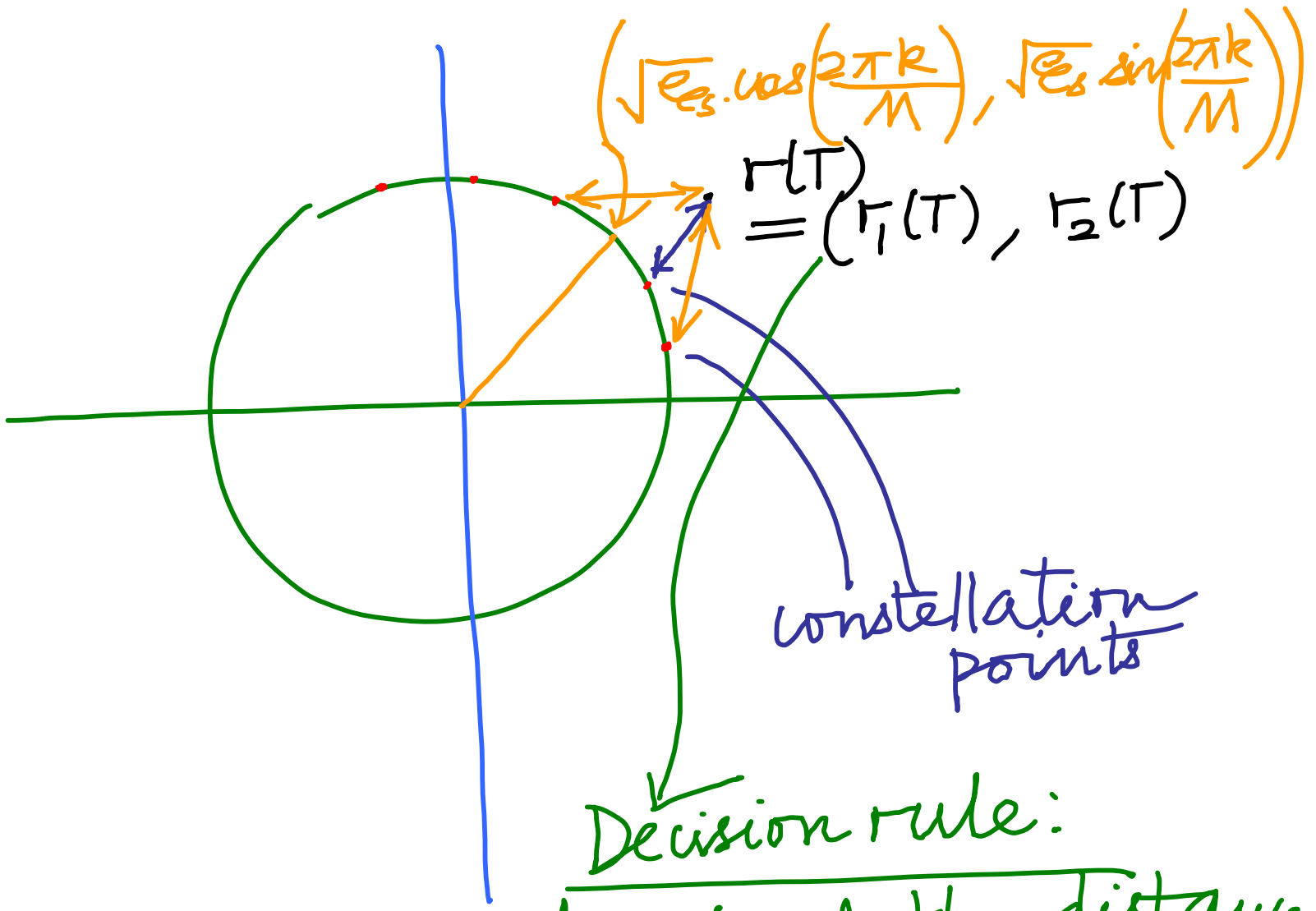
MF $h_1(t) = p_1(T-t)$

MF $h_2(t) = p_2(T-t)$.

$a_1 \mathcal{E}_p + \tilde{n}_1 = a_1 + \tilde{n}_1$ → $r_1(T)$

$a_2 + \tilde{n}_2$ → $r_2(T)$.

Gaussian mean = 0 var = $\frac{N_0}{2}$


$\left(\sqrt{E_s}\cos\left(\frac{2\pi k}{M}\right), \sqrt{E_s}\sin\left(\frac{2\pi k}{M}\right)\right)$
$r(T)$
$= (r_1(T), r_2(T))$
constellation points
Decision rule:
Look at the distance to all constellation pts and choose one with minimum distance.

S_k ← k^{th} point of M-ary PSK

$\min_k \| r(T) - S_k \|$ ← Norm Distance

Choose constellation pt k such that distance is minimum

$$r(T) = \left(r_1(T), r_2(T) \right)$$

$$S_k = \left(\sqrt{E_s} \cos\left(\frac{2\pi k}{M}\right), \sqrt{E_s} \sin\left(\frac{2\pi k}{M}\right) \right)$$

distance

$$= \min_{k} \sqrt{\left(r_1(T) - \sqrt{E_s}\cos\left(\frac{2\pi k}{M}\right)\right)^2 + \left(r_2(T) - \sqrt{E_s}\sin\left(\frac{2\pi k}{M}\right)\right)^2}$$

distance between output Test statistic $r(T)$ and k^{th} pt of M-ary PSK.

Nearest Neighbor Decoding Rule.

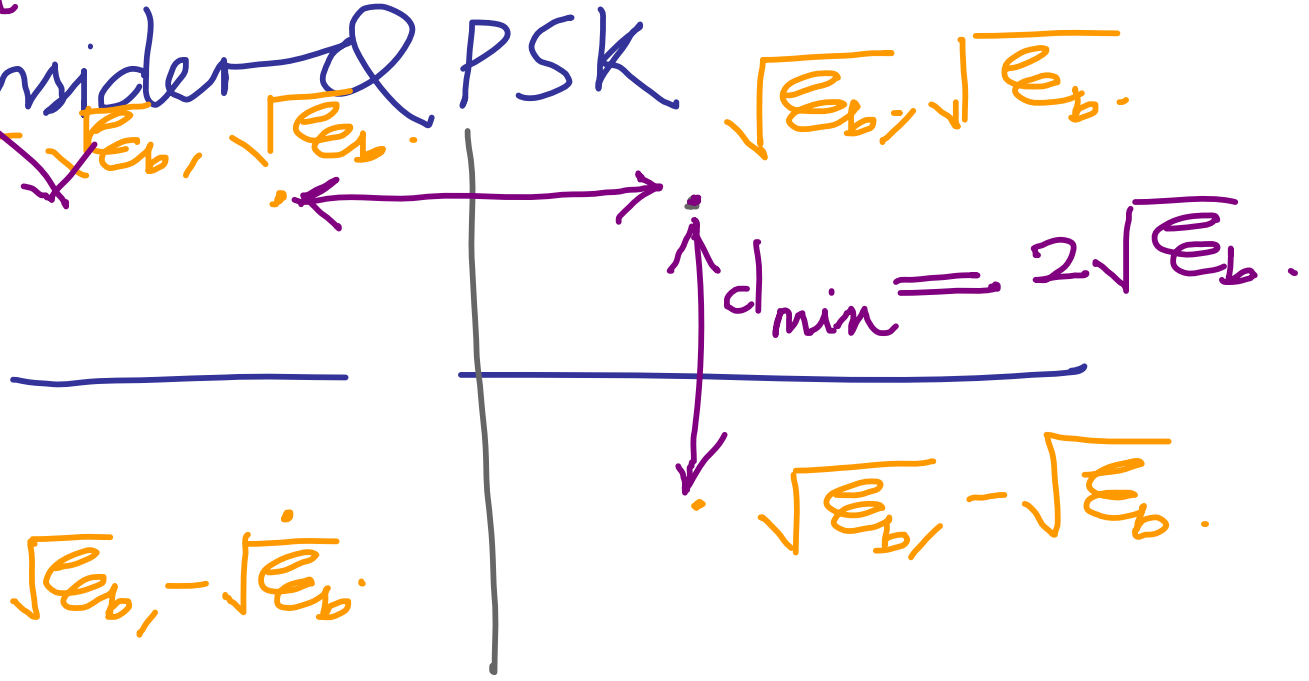
M-ary PSK BER:

We will employ an indirect approach for the BER.

Each constellation pt has 2 Nearest Neighbors

Consider PSK

$\alpha = 2$



Approx Symbol Error Rate (SER)

$$= \alpha \, Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right)$$

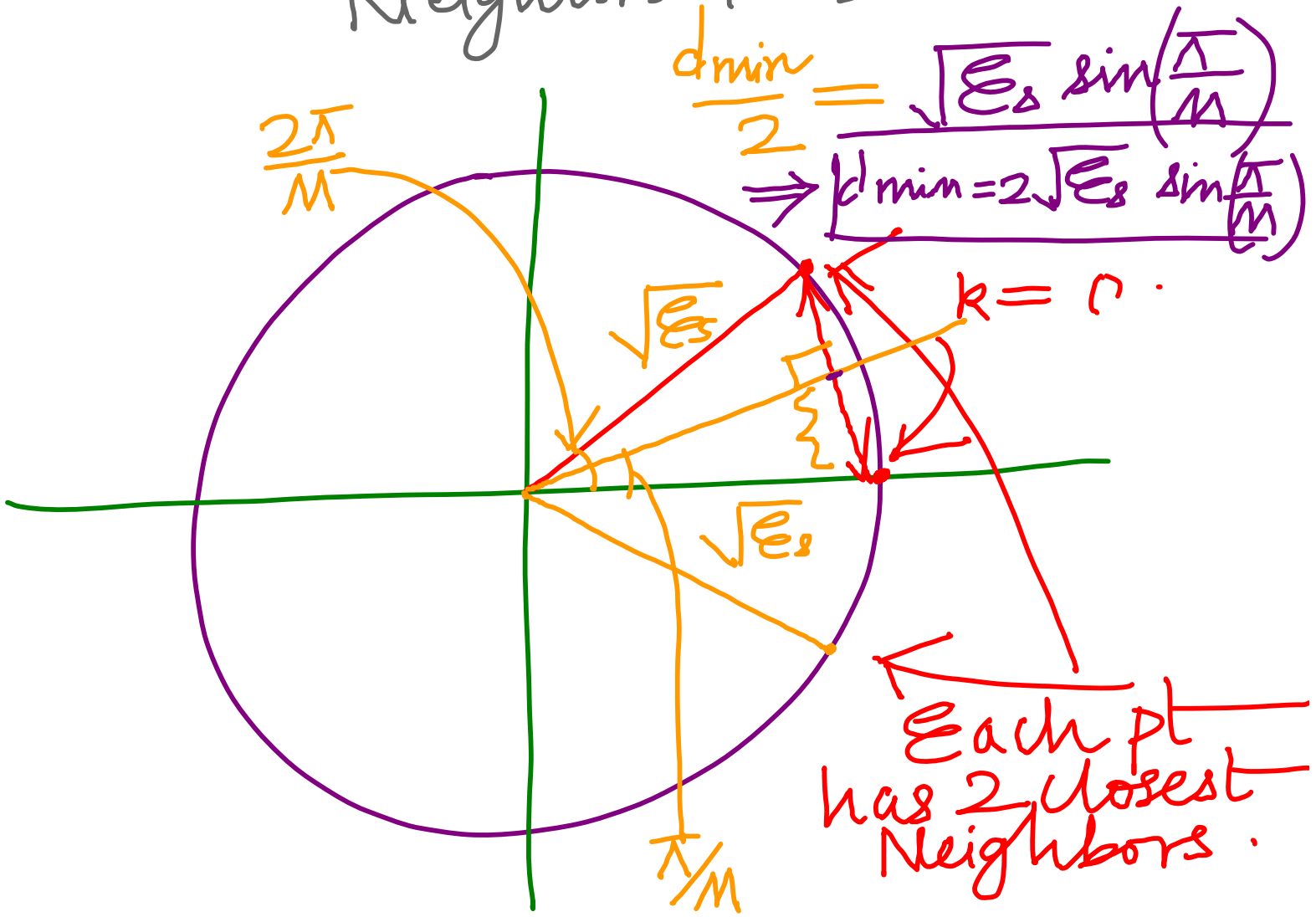
α = # Nearest Neighbors.

d_{min} = distance to Nearest Neighbor

very useful expression to compute error rate of Digital comm when exact expression is difficult to obtain

M-ary PSK:

Number of Nearest Neighbors $\alpha = 2$



d_{min} for M-ary PSK

$$d_{min} = 2\sqrt{\mathcal{E}_s}\sin\left(\frac{\pi}{M}\right)$$

$$P_e = 2Q\left(\frac{2\sqrt{\mathcal{E}_s}\sin\left(\frac{\pi}{M}\right)}{\sqrt{2N_0}}\right)$$

$$P_e = 2Q\left(\sqrt{\frac{2\mathcal{E}_s\sin^2\left(\frac{\pi}{M}\right)}{N_0}}\right)$$

Approximation for SER of M-ary PSK

Approximation is very close to actual SER at high $\frac{E_s}{N_0}$.